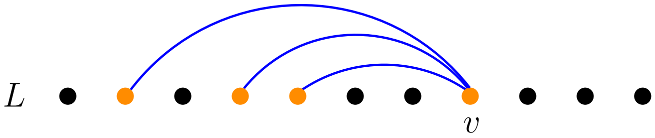
Generalized Coloring Numbers

Gwenaël Joret

Order & Geometry, 2018

Coloring number

Ordering L of $V(G)$:



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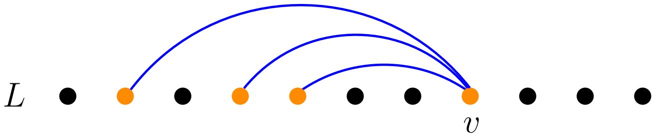

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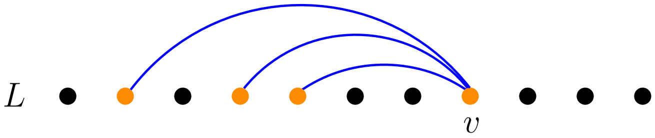

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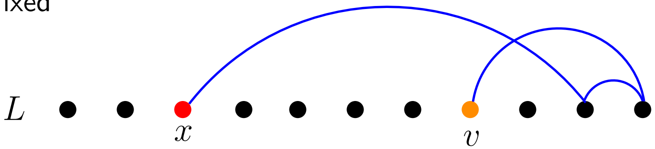
$\chi(G) \leqslant \mathrm{col}(G)$

r-Strong coloring number

$r \geqslant 1$ fixed



v strongly reaches x if $\exists$ vx-path P of length $\leqslant r$ with x left of v and all inner vertices to the right of v

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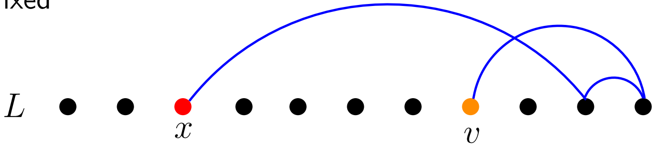

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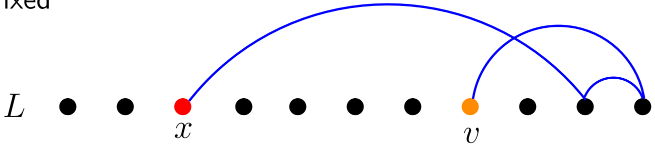

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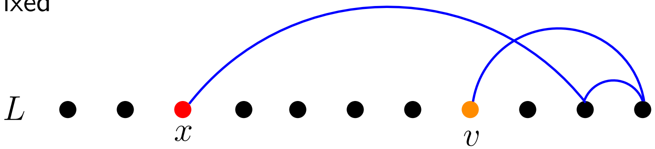

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$$\mathrm{col}(G) = \mathrm{scol}_1(G) \leqslant \mathrm{scol}_2(G) \leqslant \cdots \leqslant \mathrm{scol}_\infty(G) = \mathrm{tw}(G) + 1$$

where $\mathrm{tw}(G)$:= treewidth of G

r-Weak coloring number

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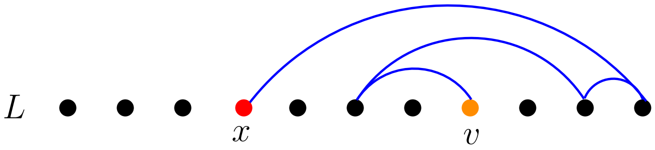

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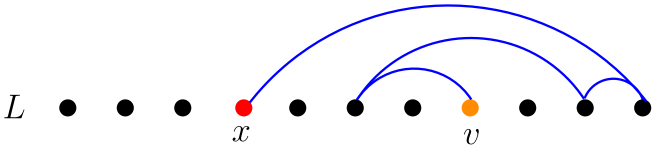

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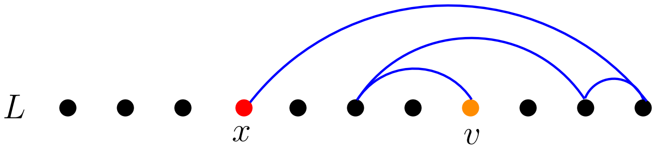

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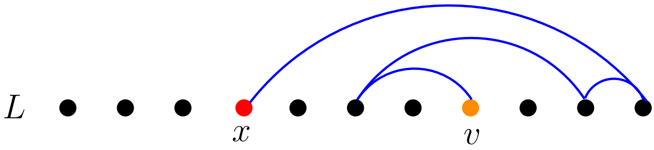

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$$\mathrm{scol}_r(G) \leqslant \mathrm{wcol}_r(G) \leqslant (\mathrm{scol}_r(G))^r$$ (Kierstead & Yang '03)

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"While our main motivation is the study of game chromatic number, there have been other applications of these ideas and we expect there will be more." (Kierstead & Yang '03, from abstract)

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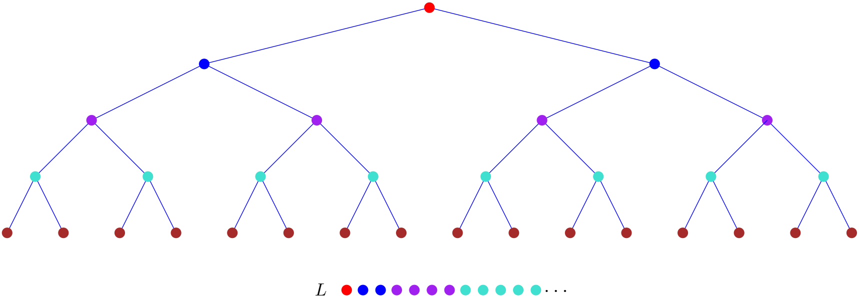
This talk: focus mostly on weak coloring numbers

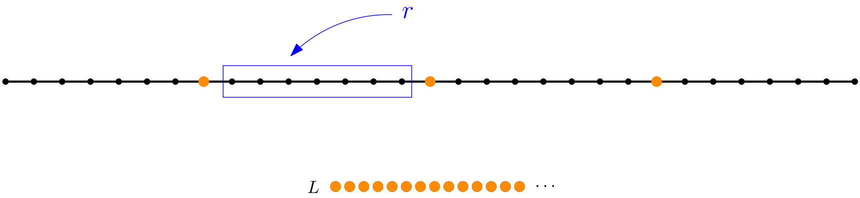
Trees



$\mathrm{wcol}_r(G) \leqslant r + 1$

Paths

Paths

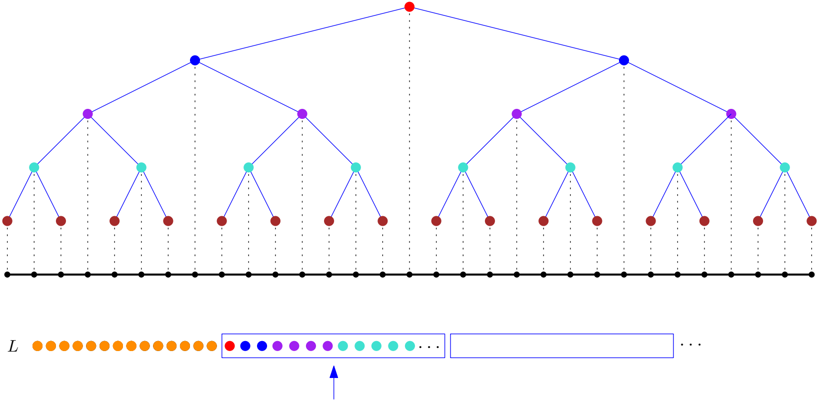


every vertex can reach at most 2 orange vertices

no vertex can reach two vertices from distinct sections

path on r vertices

path on r vertices

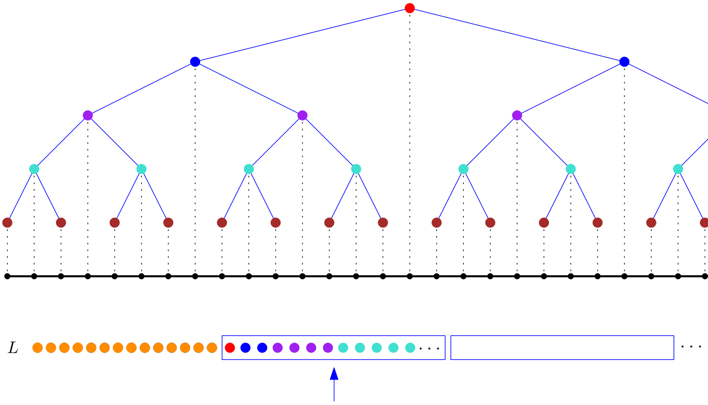


BFS ordering

every vertex reaches $\leqslant 1$ vertex of depth d $\forall d$

$\Rightarrow \quad \mathrm{wcol}_r(G) \leqslant \log r + O(1)$

path on r vertices



BFS ordering

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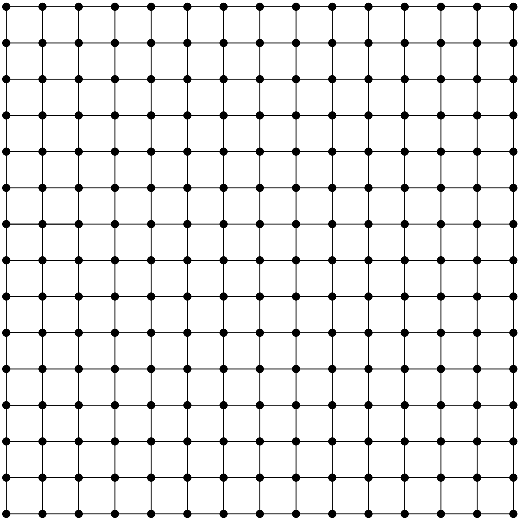
$\mathrm{wcol}_r(G) \geqslant \log r$

Grids



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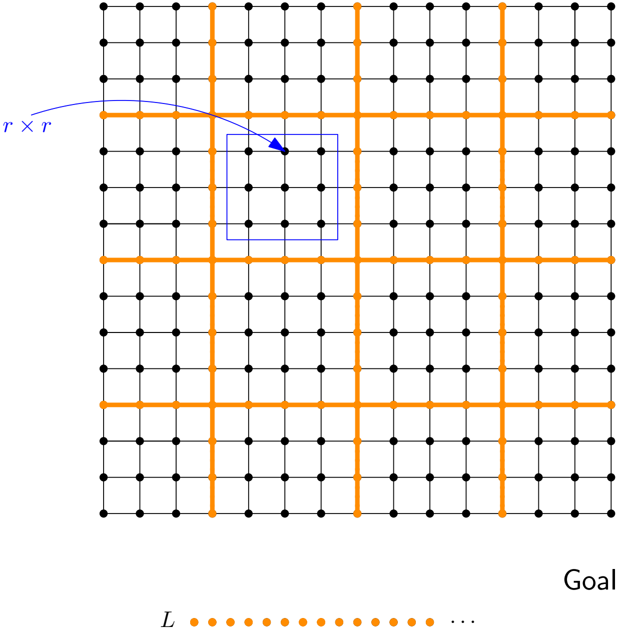
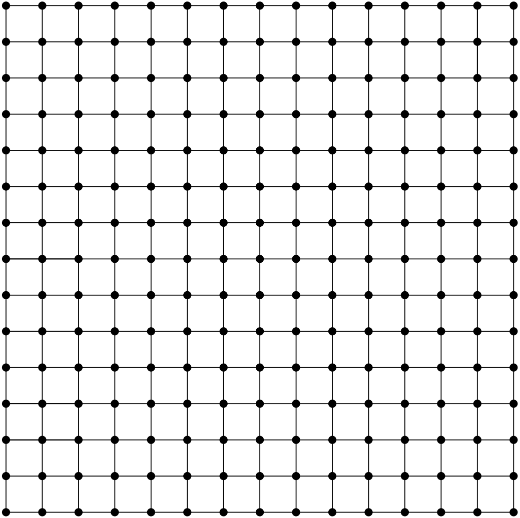


Goal: $\mathrm{wcol}_r(G) \in O(r)$

$r \times r$ grid



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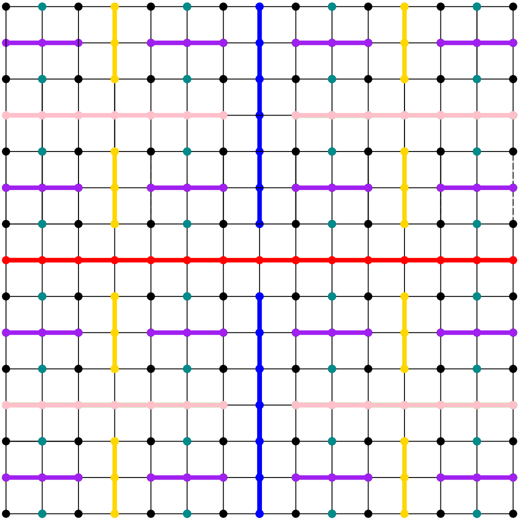

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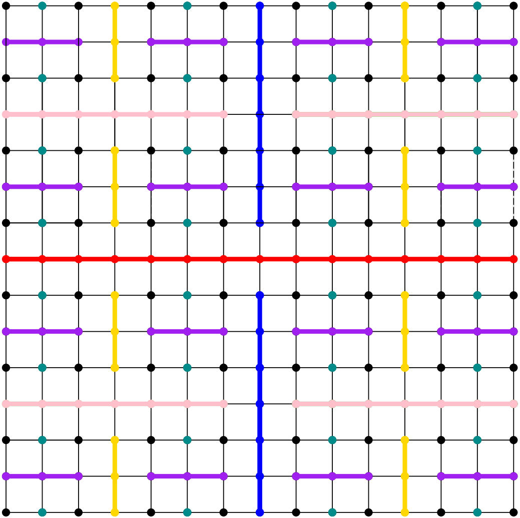


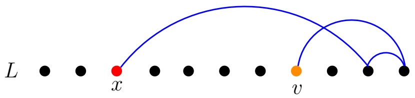
$\mathrm{wcol}_r(G) \in O(r)$ best possible

Planar graphs

Quick proof that weak/strong coloring numbers are bounded

(Dvořak, unpublished)

Focus on $\mathrm{scol}_r(G)$



Kissing coins representation, coins in nonincreasing order of radius

$\Rightarrow \mathrm{scol}_r(G) \in O(r^2)$

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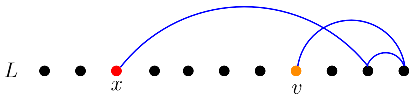

Kissing coins representation, coins in nonincreasing order of radius

$\Rightarrow$ $\mathrm{scol}_r(G) \in O(r^2)$

In fact, $\mathrm{scol}_r(G) \leqslant 5r + 1$

(van den Heuvel, Ossona de Mendez, Quiroz, Rabinovich, Siebertz '16)

Treewidth t

$$\mathrm{wcol}_r(G) \leqslant \binom{r+t}{t} \quad \in O(r^t)$$

(Grohe, Kreutzer, Rabinovich, Siebertz, Stavropoulos '16)

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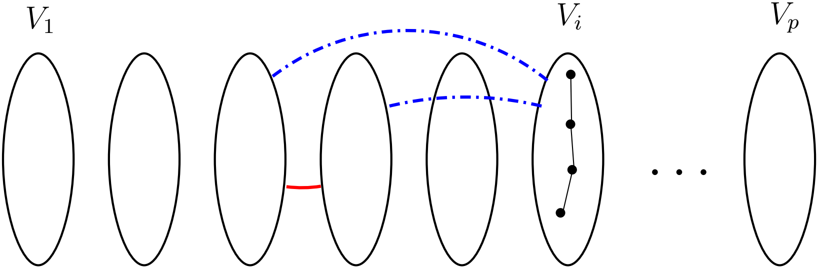
Proof: $\binom{r+t}{t} = \binom{r+t-1}{t-1} + \binom{r-1+t}{t}$

Planar graphs (again)

$$\mathrm{wcol}_r(G) \leqslant \binom{r+2}{2}(2r+1) \quad \in O(r^3)$$

(van den Heuvel, Ossona de Mendez, Quiroz, Rabinovich, Siebertz '16)

Chordal partitions



f-chordal partition: Partition $V_1, V_2, \dots, V_p$ of $V(G)$ s.t.

- $G[V_i]$ connected
- all V_j's with $j < i$ seen by V_i are pairwise adjacent
- ℓ-balls in $G[V_i \cup \dots \cup V_p]$ intersect V_i in $\leqslant f(\ell)$ vertices $\forall \ell \in \{1, \dots, r\}$

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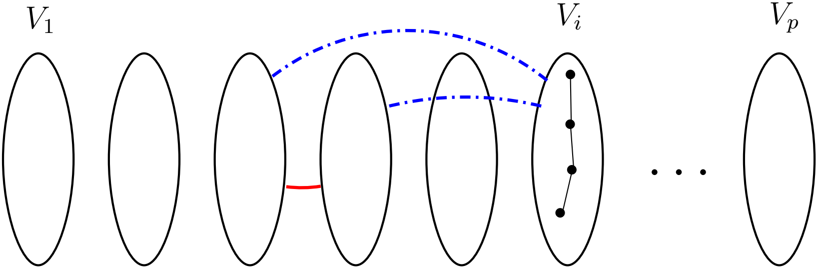

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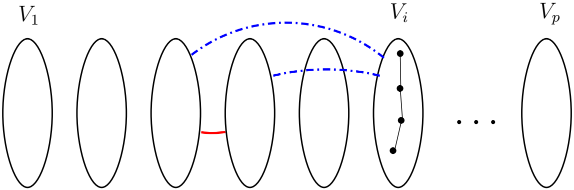
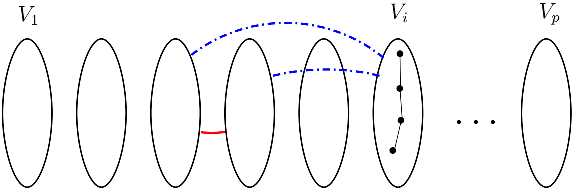
Width $:=$ Max$_k$ s.t. some V_i sees k sets V_j's with $j < i$



f-chordal partition of width k

$\Rightarrow \quad \mathrm{tw}(H) = k$ for quotient graph H



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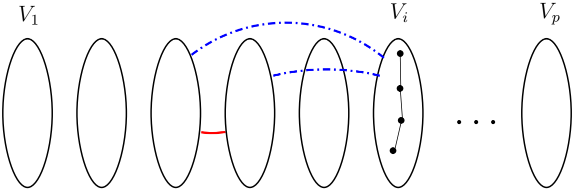
$\Rightarrow \quad \mathrm{wcol}_r(G) \leqslant \binom{r+k}{k} f(r) \quad \in O(r^k \cdot f(r))$

(van den Heuvel, Ossona de Mendez, Quiroz, Rabinovich, Siebertz '16)



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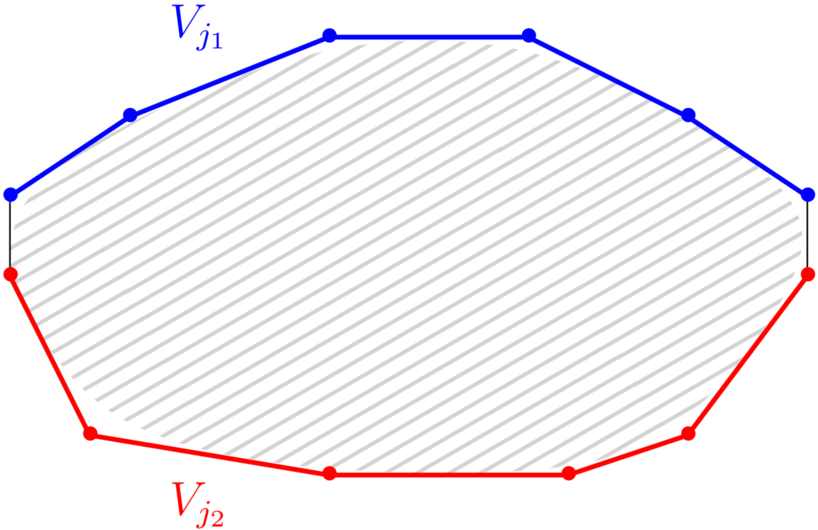
(van den Heuvel, Ossona de Mendez, Quiroz, Rabinovich, Siebertz '16)

Goal: Planar triangulations have f-chordal partitions of width k

for $k = 2$ and $f(r) = 2r + 1$

Chordal partitions of triangulations

Iteratively take a shortest path inside a region



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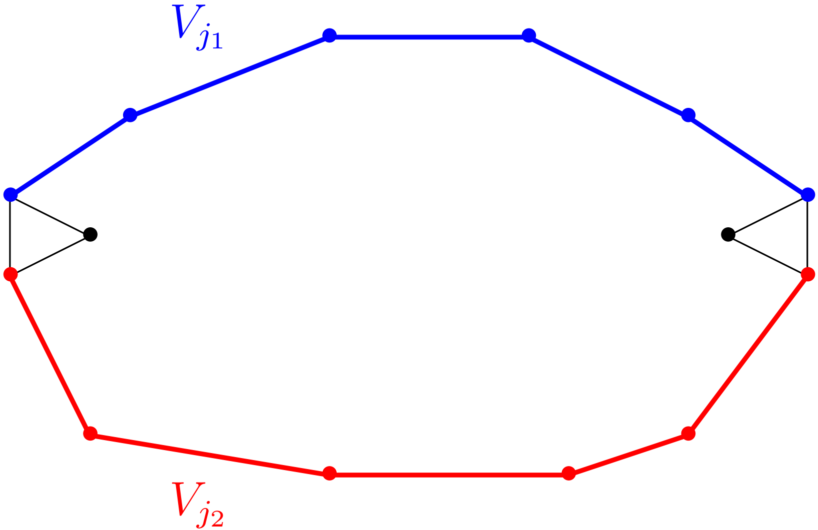

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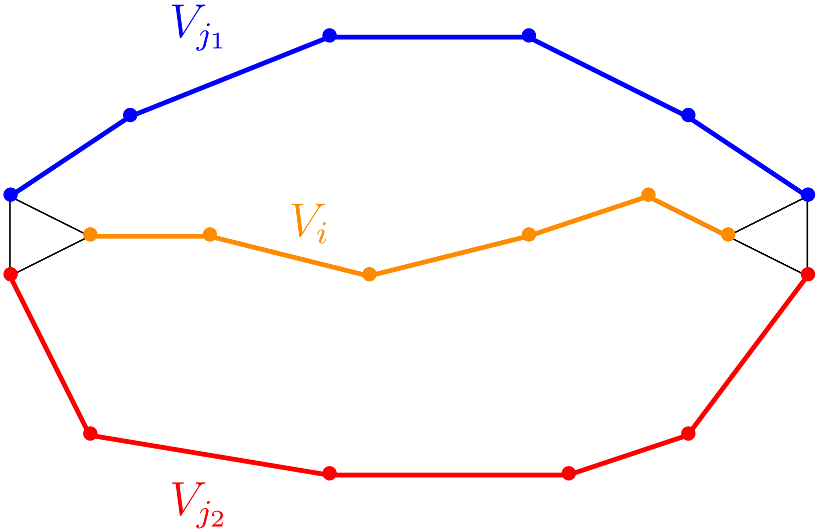

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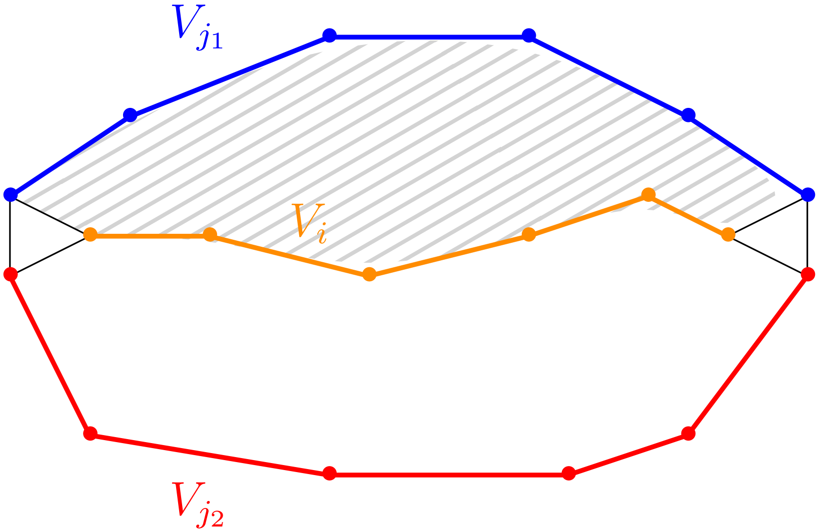

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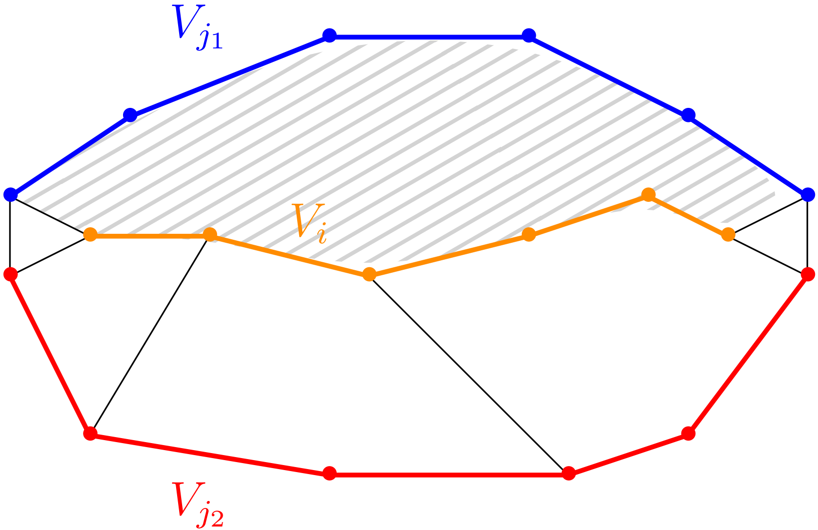

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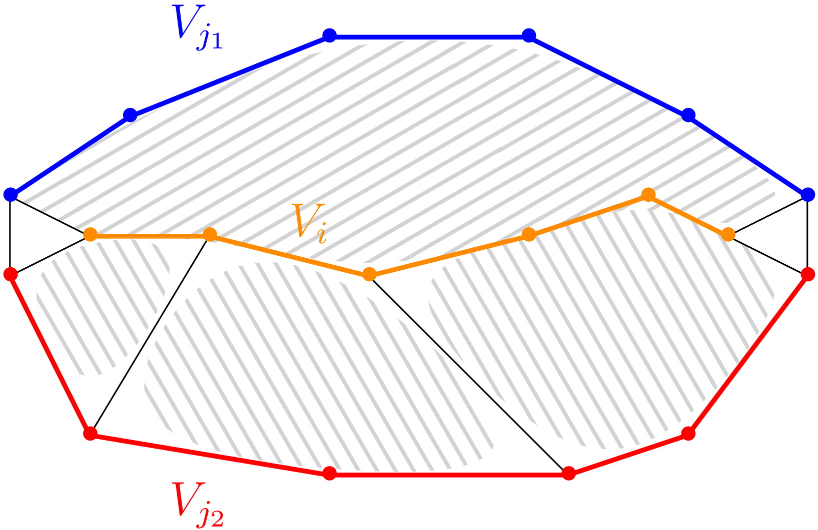

Planar graphs

$$\mathrm{wcol}_r(G) \leqslant \binom{r+2}{2}(2r+1) \quad \in O(r^3)$$

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Conjecture: $\mathrm{wcol}_r(G) \in O(r^2 \log r)$ (J., Micek '18+)

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True if also $\mathrm{tw}(G) \leqslant 3$ (stacked triangulations) (J., Micek '18+)

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If G outerplanar then:

- $\mathrm{wcol}_r(G) \in O(r^2)$ since $\mathrm{tw}(G) \leqslant 2$

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- $\mathrm{wcol}_r(G) \in O(r \log r)$ (J., Micek '18+)

$\mathrm{stw}(G) :=$ simple treewidth of G (Knauer & Ueckerdt '12)

$\mathrm{wcol}_r(G) \in O(r^{t-1} \log r) \quad$ if $\mathrm{stw}(G) = t$ (J., Micek '18+)

K_t-minor free graphs

$$\mathrm{wcol}_r(G) \leqslant \binom{r+t-2}{t-2}(t-3)(2r+1) \quad \in O(r^{t-1})$$

(van den Heuvel, Ossona de Mendez, Quiroz, Rabinovich, Siebertz '16)

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Recall: f-chordal partition of width k

$$\Rightarrow \quad \mathrm{wcol}_r(G) \leqslant \binom{r+k}{k} f(r)$$

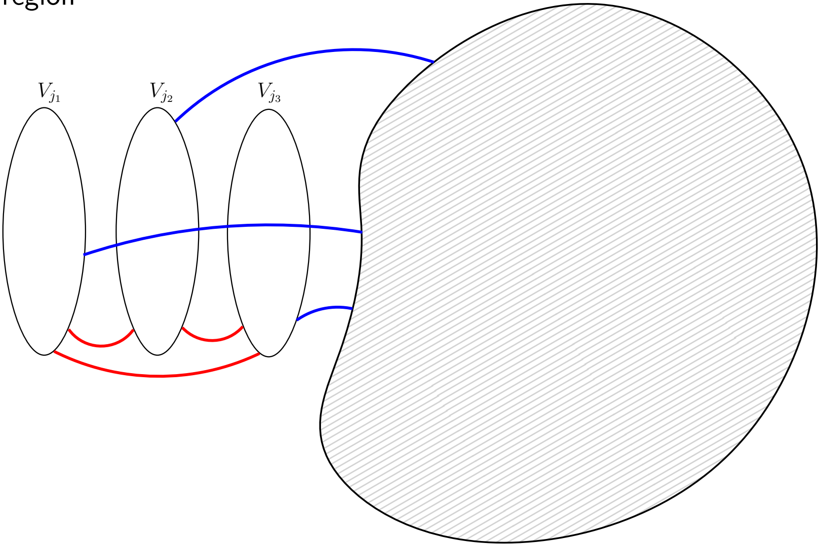
(van den Heuvel, Ossona de Mendez, Quiroz, Rabinovich, Siebertz '16)

Goal: K_t-minor free graphs have f-chordal partition of width k

for $k = t - 2$ and $f(r) = (t-3)(2r+1)$

Chordal partition of K_t-minor free graphs

Iteratively take union of $\leqslant t$ shortest paths to exit points inside a region



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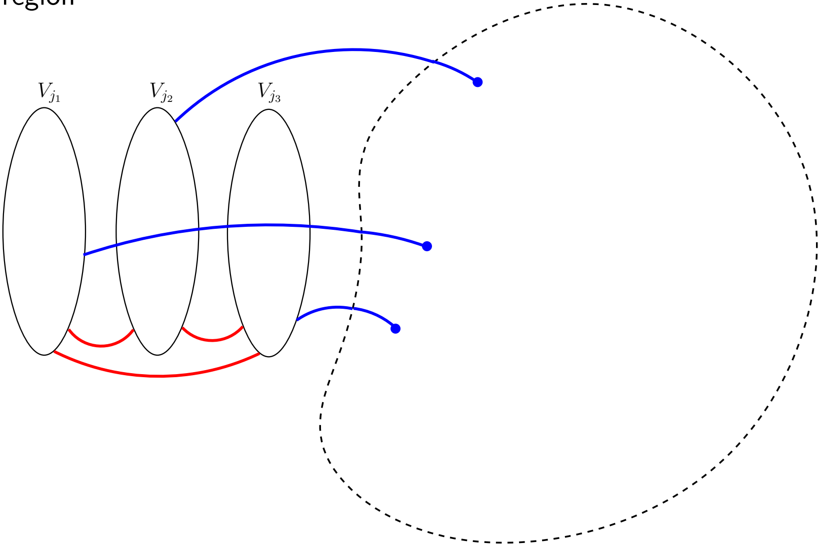

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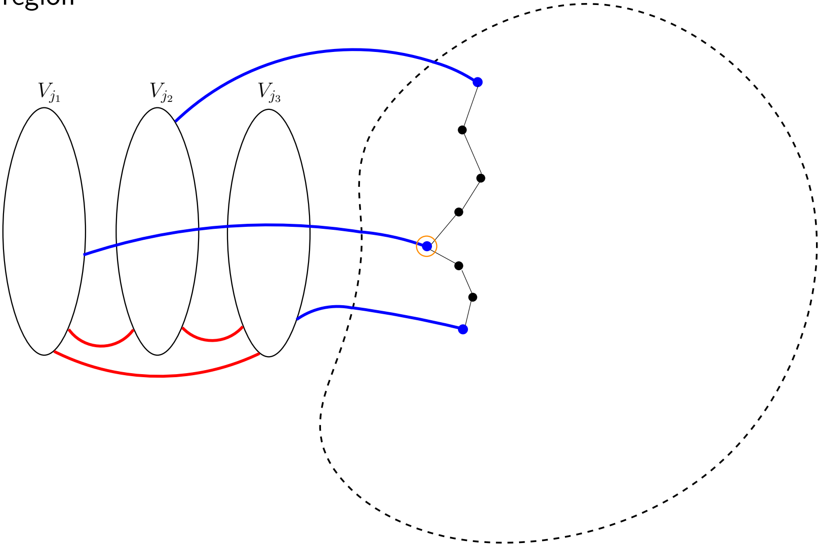

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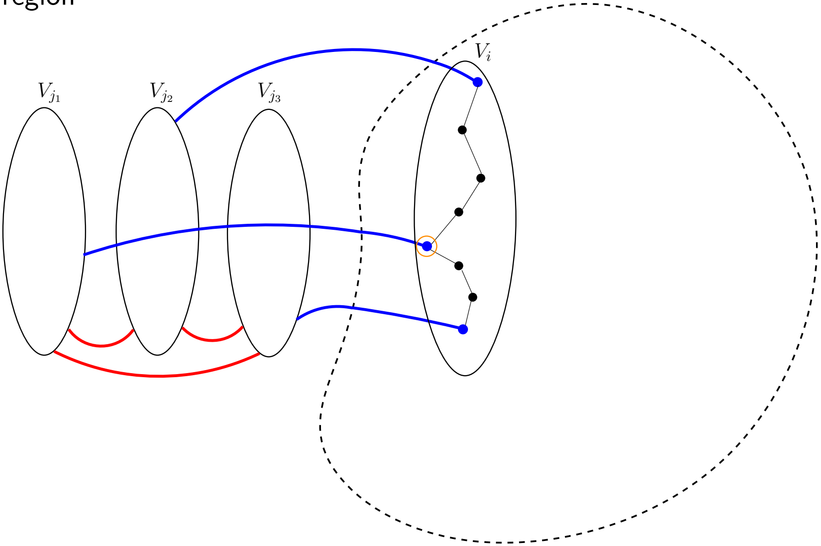

K_t-minor free graphs

$$\mathrm{wcol}_r(G) \leqslant \binom{r+t-2}{t-2}(t-3)(2r+1) \quad \in O(r^{t-1})$$

(van den Heuvel, Ossona de Mendez, Quiroz, Rabinovich, Siebertz '16)

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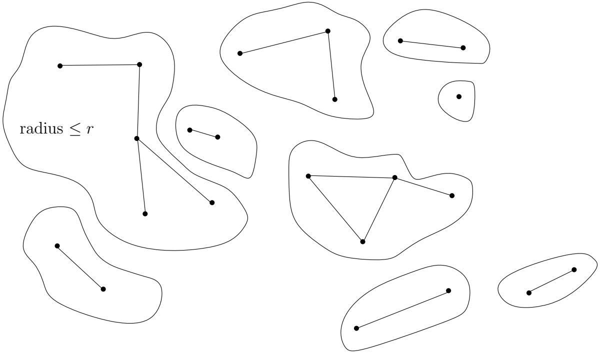
Conjecture: $\mathrm{wcol}_r(G) \in O(r^{t-2})$ (J., Micek '18+)

K_t-topological minor free

$\mathrm{wcol}_r(G) \leqslant 2^{O(r \log r)}$ (Kreutzer, Pilipczuk, Rabinovich, Siebertz '16)

Classes with bounded expansion

r-shallow minor



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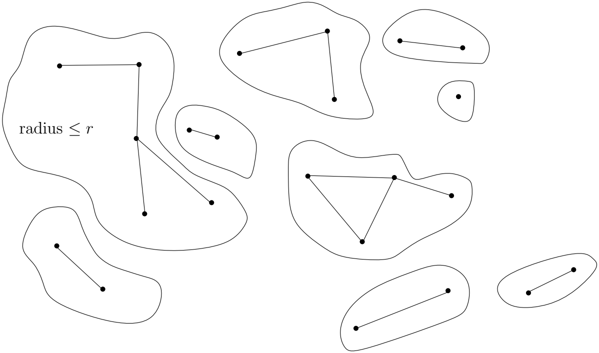

Class $\mathcal{C}$ has bounded expansion if $\exists f : \mathbb{N} \to \mathbb{R}$ s.t. all r-shallow minors of graph G have average degree $\leqslant f(r) \quad \forall r, \forall G \in \mathcal{C}$

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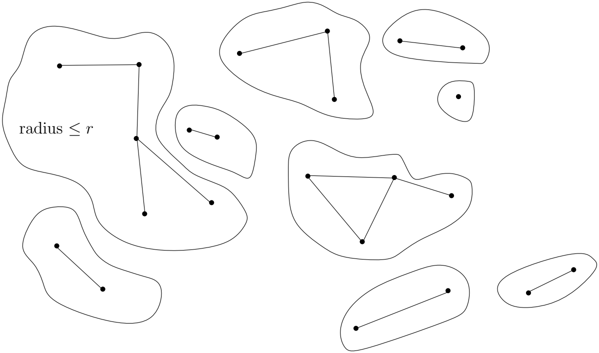

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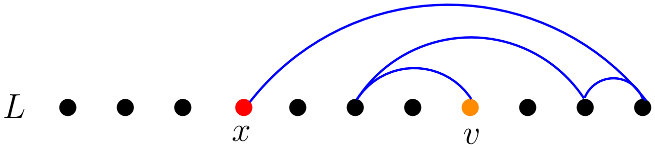
$\Leftrightarrow \exists f : \mathbb{N} \to \mathbb{R}$ s.t. $\mathrm{wcol}_r(G) \leqslant f(r) \qquad \forall r, \forall G \in \mathcal{C}$ (Zhu '09)

Universal ordering

$\mathcal{C}$:= class with bounded expansion



∃ single ordering of vertices witnessing all weak coloring numbers are bounded

(van den Heuvel & Kierstead '18+)

Formally:

$\exists f$ s.t. $\forall G \in \mathcal{C}$ $\exists L$ s.t. $|W_r^L(v)| \leqslant f(r)$ $\forall r$ $\forall v \in V(G)$

Nowhere dense classes

Class $\mathcal{C}$ is nowhere dense if $\forall \varepsilon > 0 \;\; \forall r$:

all r-shallow minors of G have average degree $O(|G|^{\varepsilon}) \quad \forall G \in \mathcal{C}$

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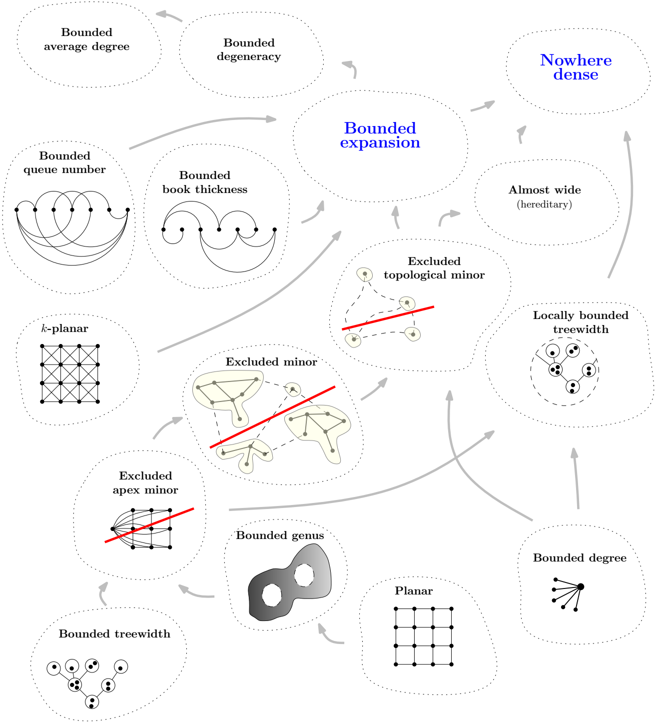
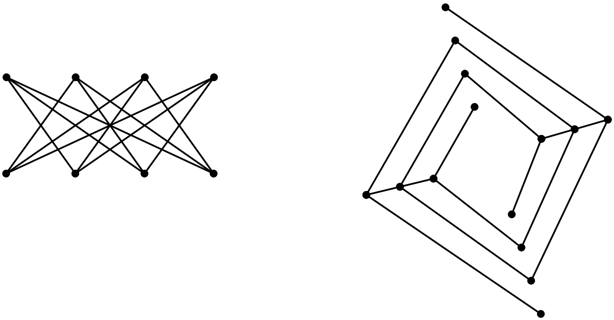


Bounded average degree
Bounded degeneracy
Nowhere dense
Bounded expansion
Bounded queue number
Bounded book thickness
Almost wide (hereditary)
Excluded topological minor
k-planar
Locally bounded treewidth
Excluded minor
Excluded apex minor
Bounded genus
Bounded degree
Planar
Bounded treewidth

Order & Geometry?

Application: Poset dimension



If P poset of height h and G cover graph of P then

$\dim(P) \leqslant 4^{\mathrm{wcol}_{3h-3}(G)}$ (J., Micek, Ossona de Mendez, Wiechert '18)

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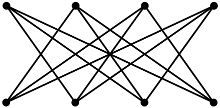

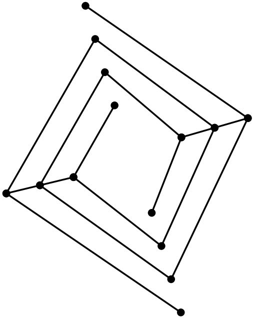

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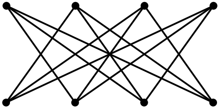

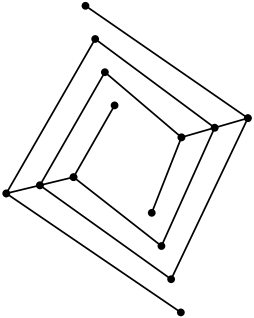

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Corollary: P of bounded height and $G \in$ nowhere dense class
$\Rightarrow \forall \varepsilon > 0 \ \dim(P) \in O(|P|^{\varepsilon})$

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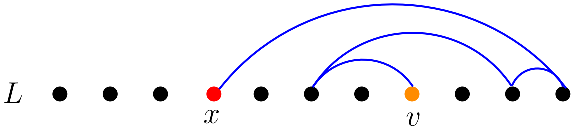

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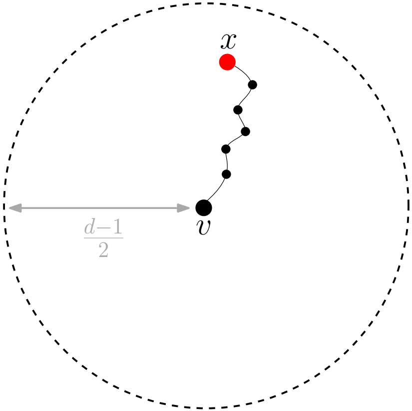

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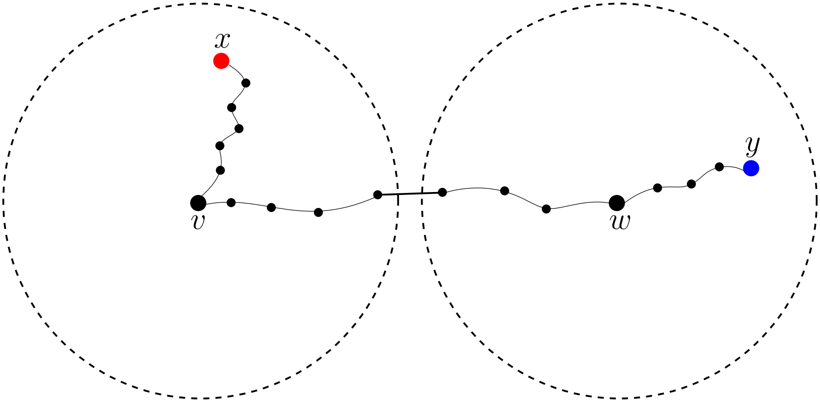
If v, w at distance exactly d then $\phi(v) \neq \phi(w)$:



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False: $\exists G$ planar with $\chi(G^{[\#d]}) \in \Omega(d/\log d)$

(Bousquet, Esperet, Harutyunyan, de Joannis de Verclos '17)

Thank you!