A Tribute to Order and Geometry Workshops

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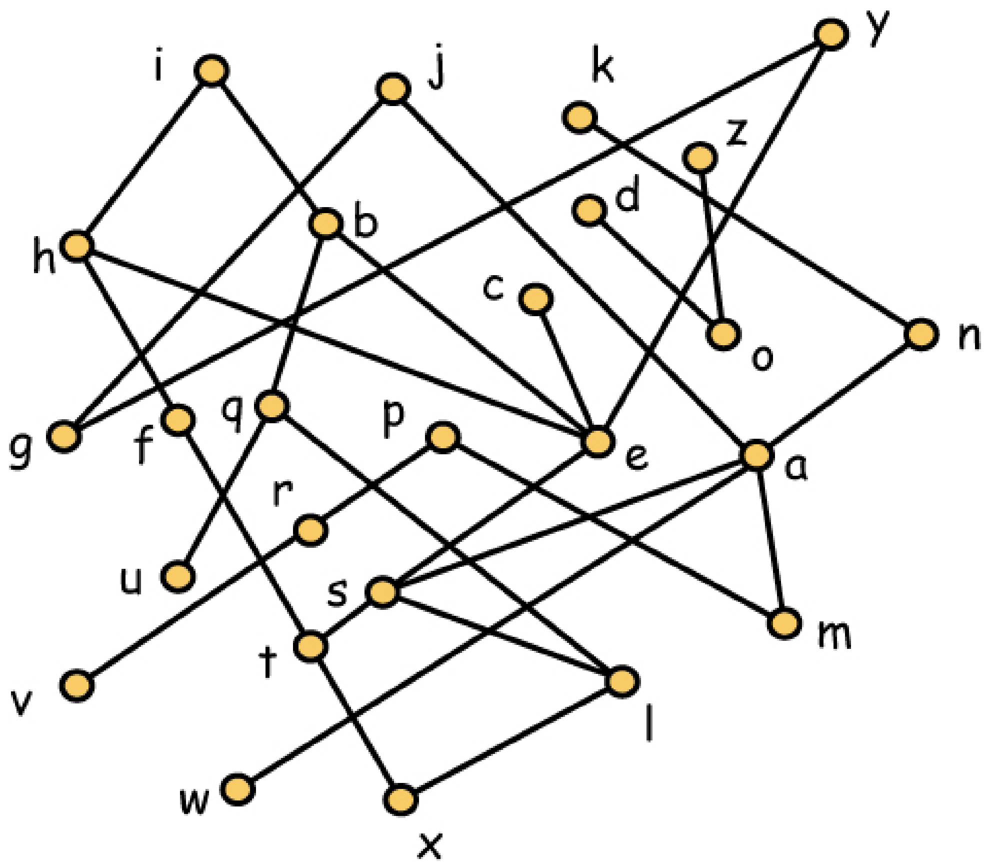
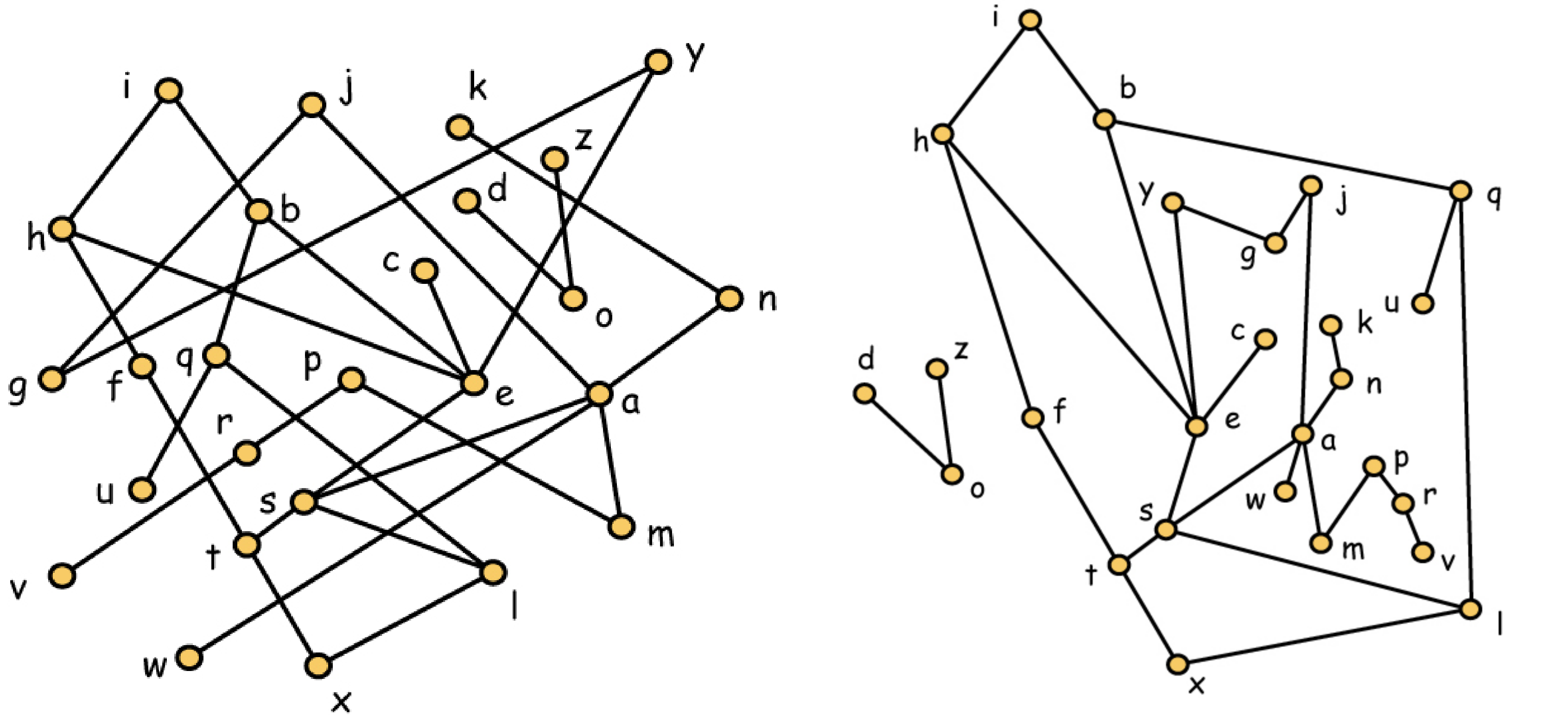
A Planar Poset on 26 points



Yes!! It's Planar!!



Components and Dimension

Fact If P is a disconnected poset and the maximum dimension of a component of P is d, then $\dim(P) = d$ unless $d = 1$, in which case $\dim(P) = 2$.

Fact If P is a disconnected poset and the maximum local dimension of a component of P is d, then

$$\mathrm{ldim}(P) \leq d + 2.$$

Observation The analogous problem for Boolean dimension is more subtle, since Boolean realizers for the components can use different truth functions.

Components and Boolean Dimension

Theorem (Mészáros, Micek and WTT, '18+) If P is a disconnected poset and the maximum Boolean dimension of a component of P is d, then

$$\mathrm{bdim}(P) \leq 2 + d + 4 \cdot 2^d.$$

Furthermore, there is a disconnected poset P with Boolean dimension $\Omega(2^d/d)$ such that every component has Boolean dimension at most d.

Blocks and Dimension

Observation It is not immediately clear that there is any bound of the dimension of a connected poset when every block has dimension at most d.

Theorem (WTT, Walczak and Wang, '18) If P is a connected poset and the maximum dimension of a block of P is d, then

$$\dim(P) \leq d + 2.$$

Furthermore, this inequality is best possible.

Blocks and Boolean Dimension

Theorem (Mészáros, Micek and WTT, '18+) If P is a connected poset and the maximum Boolean dimension of a block of P is d, then

$$\mathrm{bdim}(P) \leq 9 + d + 24 \cdot 2^d.$$

Furthermore, there is a connected poset P with Boolean dimension $\Omega(2^d/d)$ such that every block has Boolean dimension at most d.

Blocks and Local Dimension

Observation This time, the analogous question for local dimension is more complex. Indeed, it is not clear that there is any bound on the local dimension of a connected poset P when every block of P has local dimension at most d. In fact, there is no such bound!

Theorem (Bosek, Grytczuk and WTT, '18+) For every $d \geq 1$, there is a connected poset P such that $\mathrm{ldim}(P) > d$ and $\mathrm{ldim}(B) \leq 3$ for every block B of P.

Dimension and Tree-Width

Theorem (Joret, Micek, Milans, WTT, Walczak and Wang, '16) For every pair (h, t) of positive integers, there is a constant $c(h, t)$ so that if the tree-width of the cover graph of P is at most t and the height of P is at most h, then

$$\dim(P) \leq c(h, t).$$

Observation When $t \geq 3$, the Kelly construction shows that $c(h, t)$ must tend to infinity with h. We now know the growth rate must be exponential.

Boolean Dimension and Tree-Width

Theorem (Felsner, Mészáros and Micek, 18+) For every $t \geq 1$, there is a constant $c(t)$ so that if the tree-width of the cover graph of a poset P is at most t, then

$$\mathrm{bdim}(P) \leq c(t).$$

Observation The important detail here is the upper bound is independent of height.

Local Dimension and Path-Width

Theorem (Barrera-Cruz, Prag, Smith, Taylor and WTT, 18+) For every $t \geq 1$, there is a constant $c(t)$ so that if the path-width of the cover graph of a poset P is at most t, then

$$\mathrm{ldim}(P) \leq c(t).$$

Theorem (B-C, P, S, T and WTT, 18+) For every $d \geq 1$, there is a poset P such that $\mathrm{ldim}(P) > d$ and the tree-width of the cover graph of P is 3.

Local Dimension and Boolean Dimension

Theorem (Barrera-Cruz, Prag, Smith, Taylor and WTT, 18+) For every $d \geq 1$, there is a poset P such that

$$\mathrm{ldim}(P) > d \quad \text{and} \quad \mathrm{bdim}(P) = 4.$$

Theorem (WTT and Walczak, 18+) For every $d \geq 1$, there is a poset P such that

$$\mathrm{bdim}(P) > d \quad \text{and} \quad \mathrm{ldim}(P) = 4.$$

Local Dimension and Planar Posets

Theorem (Bosek, Grytczuk and WTT, '18+) For every $d \geq 1$, there is a planar poset P such that $\mathrm{ldim}(P) > d$.

Theorem (Barrera-Cruz and Smith, '18+) The posets in the construction used to prove this theorem have Boolean dimension at most 7.

A Very Big Thank You to (since 2016):

1. Stefan and Piotrek.
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